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Theory Matters for Financial Advice!

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Theory Matters for Financial Advice!*

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Abstract

We show that the optimal asset allocation for an investor depends crucially on the theory with which the investor is modeled. For the same market data and the same client data different theories lead to different portfolios. The market data we consider is standard asset allocation data. The client data is determined by a standard risk profiling question and the theories we apply are mean–variance analysis, expected utility analysis and cumulative prospect theory.

Keywords: Cumulative Prospect Theory, Mean–Variance Analysis.

JEL–classification: C61, D81, G02, G11.

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1 Introduction

Financial advice is one of the most important services offered in the financial industry. A financial advisor needs to know which trade-off markets deliver and which risk tolerance a client has so that he can recommend to him a portfolio of assets that is optimally placed on the financial markets' trade-off. Thus the role of a financial advisor is to build a bridge between the clients and the markets. Using a decision theory is a huge simplification in this task. A decision theory structures the market data along simple key characteristics (e.g. means and variances of returns) and it suggests a way to model the risk tolerance of a client (e.g. by variance aversion). Without a decision theory the financial advisor would have to explain to the client all characteristics of the markets for all possible portfolios he can recommend and then infer the client's risk tolerance in all those dimensions. It is clear that such an approach is impossible both because the time and the capabilities of the client (and the advisor) are limited.

In this paper we show that the choice of the decision theory is not innocuous for the financial advice given. For the same data on the markets and the same clients the recommended portfolios differ substantially with the decision theory chosen. The decision theories we compare are mean-variance theory, expected utility theory and cumulative prospect theory. Thus we cover a broad range of decision theories used by practitioners, academics preferring rational and those preferring behavioral decision theories. Depending on the market data the median differences (measured in terms of certainty equivalents) can be as high as 6.5% p.a.! That is to say, if the advisor assumes the client is best described by one of the three decision models but the client is actually a decision maker who follows a different decision model then the recommended portfolio can have a return that is 6.5% lower p.a. than the portfolio that is optimal for the client.

To substantiate our claim we combine market data that is standard for asset allocation purposes data with a collection of agents assessed by Abdellaoui et al. (2007) [1] according to Cumulative Prospect Theory. Then we map the clients into other decision models by figuring out the answer to a standard risk tolerance question that a CPT-client with the characteristics found in Abdellaoui et al. gives. This answer is then interpreted as an answer of the client if he were of a different decision model type (mean-variance, expected utility). Notice that a financial advisor always sees the same answer given by the client to this standard risk tolerance question. Whether the financial advisor interprets this answer according to CPT, mean-variance or expected

utility depends on the decision theory he uses. For these theories we then compute the optimal portfolios on the market data.

Our approach is novel but it should be compared with Levy and Levy (2004) [12] who compare optimal CPT-portfolios computed in two different ways: CPT restricted to the mean–variance efficient frontier and CPT without such restriction. In Levy and Levy (2004) [12] the same decision model (CPT) is assumed and two different solution techniques for finding the optimal CPT-portfolio are considered. The finding of Levy and Levy (2004) is that these different solution techniques do not matter much. De Giorgi and Hens (2009) [4] for original prospect theory (PT) and Hens and Mayer (2012) [8] for CPT extend Levy and Levy (2004) to realistic data sets in which case the differences become larger. In our paper we find even larger differences because we do not only change the optimization technique for a given decision model but we also change the decision theory with which we model the decision maker.

The rest of the paper is organized as follows: In Section 2 we first briefly discuss the objective function of CPT, followed by the formulation of the four different portfolio selection models considered in this paper. Subsequently we discuss the numerical solution aspects and present the proximity measure utilized for comparing the different optimal portfolios. Section 3 first describes and discusses the data sets that are utilized in our numerical experiments, followed by the presentation of our method for determining the risk–aversion coefficients for the mean–variance, quadratic utility and power utility (CRRA) portfolio selection models. In Section 4 we present the numerical results that we obtained for the comparison of the portfolios computed by the different portfolio selection approaches. Section 5 concludes the paper.

2 Formulation of the problems

We consider portfolios consisting of n assets; ξ denotes the vector of asset returns and λ_i is the weight of the i^{th} asset in the portfolio, $i = 1, \dots, n$.

Throughout this paper, we assume that the asset returns are finitely distributed, given by a table of scenarios and corresponding probabilities:

$$\left(\begin{array}{ccc} \xi^1, & \dots, & \xi^S \\ p_S, & \dots, & p_S \end{array} \right); \quad p_s > 0, \forall s \quad \text{and} \quad \sum_{s=1}^S p_s = 1.$$

2.1 The Cumulative Prospect Theory objective function

In this section we present a short introduction to the main aspects of Cumulative Prospect Theory (CPT). For a detailed presentation of prospect theory see, e.g., Hens and Bachmann (2008) [7] or Wakker (2010) [18]; for an integrated presentation of MV, CAPM and CPT, see Levy (2012) [11].

As a first step, we introduce a value function that plays a role analogous to that of a utility function in expected utility theory. We will employ the piecewise power value function of Tversky and Kahneman (1992) [16], which can be formulated as follows:

$$v(x) = \begin{cases} (x - RP)^{\alpha^+}, & \text{if } x \geq RP \\ -\beta(RP - x)^{\alpha^-}, & \text{if } x < RP \end{cases}, \quad (1)$$

where RP is the reference point, α^+ and α^- are the risk aversion parameters and β denotes the loss aversion parameter. The value function (1) is consistent with the theory developed in Tversky and Kahneman (1992) [16] only if the conditions $0 < \alpha^+, \alpha^- \leq 1$ and $\beta > 1$ are assumed to hold. Tversky and Kahneman (1992) [16] found that the median parameter values are $\alpha^+ = \alpha^- = 0.88$ and $\beta = 2.25$. According to the value function (1), investors evaluate their gains and losses with respect to a reference point RP . In the gain domain, investors are risk averse, whereas in the loss domain, risk-seeking behavior prevails. The function is steeper in the loss domain than in the gain domain (provided that $\alpha^+ = \alpha^-$ holds) due to the requirement that $\beta > 1$.

In the original prospect theory (PT) of Kahneman and Tversky (1979) [10], the objective function V_{PT} is formulated in a similar manner as the formulation of expected utility:

$$V_{PT}(\xi^T \lambda) = \sum_{s=1}^S w(p_i) v((\xi^s)^T \lambda). \quad (2)$$

Notice, however, that in the above formulation the probabilities are replaced by their distorted values, $p_i \mapsto w(p_i)$. The probability distortion function w models observed investor behavior by overweighing small probabilities and underweighing large probabilities. We will use the original probability weighting function of Tversky and Kahneman (1992), which can be formulated as follows:

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1 - p)^\gamma)^{\frac{1}{\gamma}}} \quad (3)$$

with $0 < \gamma \leq 1$. According to the experiments of Tversky and Kahneman (1992), the median value for γ is $\gamma = 0.65$.

In cumulative prospect theory, the probability weighting is applied to the cumulative probability distribution rather than to the individual probabilities. Consequently, to formulate the objective function for cumulative prospect theory for a fixed λ , the vector of portfolio returns $((\xi^1)^T \lambda, \dots, (\xi^S)^T \lambda)$ is first sorted in increasing order. Let $((\eta^1)^T \lambda, \dots, (\eta^S)^T \lambda)$ denote the sorted vector, for which

$$(\eta^1)^T \lambda \leq \dots \leq (\eta^t)^T \lambda \leq RP \leq (\eta^{t+1})^T \lambda \leq \dots \leq (\eta^S)^T \lambda$$

holds, and let $(\bar{p}_1, \dots, \bar{p}_S)$ be the correspondingly sorted vector of probabilities. Given these specifications, the objective function is computed according to the following equation:

$$V_{CPT}(\xi^T \lambda) = \sum_{i=1}^S \pi_i v((\eta^i)^T \lambda) \quad (4)$$

where the weights π_i are determined using the probability weighting functions w^+ and w^- :

$$\begin{aligned} \pi_1 &= w^-(\bar{p}_1) \quad \text{and} \quad \pi_i = w^-(\bar{p}_1 + \dots + \bar{p}_i) - w^-(\bar{p}_1 + \dots + \bar{p}_{i-1}) \quad \text{for } 2 \leq i \leq t; \\ \pi_S &= w^+(\bar{p}_S) \quad \text{and} \quad \pi_j = w^+(\bar{p}_j + \dots + \bar{p}_S) - w^+(\bar{p}_{j+1} + \dots + \bar{p}_S) \quad \text{for } t < j \leq S-1, \end{aligned}$$

and where the probability weighting functions are defined as follows:

$$w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}} \quad \text{and} \quad w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}} \quad (5)$$

with $0 < \delta \leq 1$ and $0 < \gamma \leq 1$. According to this construction, both tails of the probability distribution are distorted. Thus, although prospect theory overweighs small probabilities, cumulative prospect theory weighs both types of extreme events.

2.2 The portfolio selection models considered

In all portfolio selection models we exclude short sales, i.e., the constraint $\lambda \geq 0$ is imposed throughout.

The first portfolio selection problem aims at maximizing the CPT objective function V_{CPT} :

$$\left. \begin{aligned} \max_{\lambda} \quad & V_{CPT}(\xi^T \lambda) \\ & \mathbb{1}^T \lambda = 1 \\ & \lambda \geq 0, \end{aligned} \right\} \quad (6)$$

where we employed the notation $\mathbb{1}^T = (1, \dots, 1)$. The optimal solution to the above problem is denoted by λ_{CPT}^* .

In this paper we consider maximization of expected power and quadratic utility as further portfolio selection approaches and report our numerical results regarding the comparison of the optimal solutions with those from the CPT approach.

The next problem formulation is:

$$\left. \begin{aligned} \max_{\lambda} \quad & \mathbb{E}[u_Q(\xi^T \lambda)] \\ & \mathbb{1}^T \lambda = 1 \\ & \lambda \geq 0, \end{aligned} \right\} \quad (7)$$

where $u_Q(x) := x - \kappa \cdot x^2$ is a quadratic utility function with κ standing for the risk aversion parameter. The objective function can clearly be reformulated as

$$\mathbb{E}[u_Q(\xi^T \lambda)] = \mathbb{E}[\xi^T \lambda - \kappa \cdot (\xi^T \lambda)^2] = r^T \lambda - \kappa \cdot \lambda^T (\Sigma + r r^T) \lambda = r^T \lambda - \kappa \cdot \lambda^T \bar{\Sigma} \lambda$$

with $r := \mathbb{E}[\xi]$ denoting the vector of expected asset returns, Σ denoting the covariance matrix of ξ and $\bar{\Sigma} = \Sigma + r r^T$. Thus (7) can be formulated as the following quadratic optimization problem:

$$\left. \begin{aligned} \max_{\lambda} \quad & r^T \lambda - \kappa \cdot \lambda^T \bar{\Sigma} \lambda \\ & \mathbb{1}^T \lambda = 1 \\ & \lambda \geq 0. \end{aligned} \right\} \quad (8)$$

For $\kappa \geq 0$ this is clearly a convex optimization problem. The optimal solution to this problem (QEU) will be denoted by λ_Q^* .

For comparison reasons we also consider the the following risk-adjusted return formulation of the mean-variance problem:

$$\left. \begin{aligned} \max_{\lambda} \quad & r^T \lambda - \alpha \cdot \lambda^T \Sigma \lambda \\ & \mathbb{1}^T \lambda = 1 \\ & \lambda \geq 0 \end{aligned} \right\} \quad (9)$$

with the risk-aversion parameter α . The optimal solution of the above problem (MV) will be denoted by λ_{MV}^* . Notice the difference between the risk-adjusted return formulation of the MV problem and the expected quadratic utility approach: whereas in (9) we have the original covariance matrix Σ of the asset returns, in (8) a transformed covariance matrix $\bar{\Sigma}$ appears with $\bar{\Sigma} = \Sigma + rr^T$.

The fourth type of portfolio selection problems which we consider is based on CRRA utility functions. The model formulation is:

$$\left. \begin{aligned} \max_{\lambda} \quad & \mathbb{E}[u_{CRRA}(1 + \xi^T \lambda)] \\ & \mathbb{1}^T \lambda = 1 \\ & \lambda \geq 0, \end{aligned} \right\} \quad (10)$$

where $u_{CRRA}(x) = \frac{1}{\theta} x^\theta$ is a power utility function belonging to the CRRA class. The optimal solution of the above problem (CRRA) will be denoted by λ_{CRRA}^* . For $0 < \theta \leq 1$ the utility function u_{CRRA} is concave whereas for $\theta > 1$ it is convex. Notice, that the concavity of the CRRA utility function is also ensured for $\theta < 0$. For a detailed discussion of the CRRA family see Wakker (2008) [17].

2.3 The numerical solution approaches

The CPT portfolio selection problem (6) involves a non-convex and non-smooth objective function thus it is a difficult problem from the point of view of numerical solution. We apply an adaptive simplex grid method for the solution of this type of problems, as described in Hens and Mayer (2012) [8].

The quadratic optimization problems (8) and (9) are convex programming problems if $\kappa \geq 0$ and $\alpha \geq 0$ hold, respectively. In this cases we apply the standard nonlinear programming software MINOS [13] for solving them. If the above conditions do not hold, we face a problem of maximizing a convex function over a polyhedron. It is well-known, that an optimal solution of this type of problems is located on one of the vertices. In our case this means that we just have to compute the objective function in turn for the cases, when the whole wealth is invested into a single asset and choose the asset with the maximal objective value.

Analogous observations hold in the CRRA case (10) and we applied the solution approach as described for the quadratic case above.

2.4 Comparing the portfolios corresponding to the different model formulations

For comparing the optimal portfolios resulting from the different model formulations, some kind of a “similarity” or “proximity” measure is needed.

DeMiguel, Garlappi and Uppal (2009) [5] and also De Giorgi and Hens (2009) [4] utilize the difference of the certainty equivalents for assessing difference in the portfolio allocations according to PT and MV. In this paper we utilize this technique by comparing the portfolios on the basis of certainty equivalents (CE), which allows for interpreting the difference as added value in monetary terms.

For a utility function v and corresponding expected utility function V , the certainty equivalent CE_v of a portfolio λ is defined as

$$v(CE_v[\xi^T \lambda]) = V(\xi^T \lambda) \Leftrightarrow CE_v[\xi^T \lambda] = v^{-1}(V(\xi^T \lambda)).$$

Let λ_A^* be the portfolio obtained by solving an assumed model out of the models (6), (8), (9), (10), whereas λ_T^* is the solution of the true (actual) model with utility function v of the investor. Then we employ

$$CE_v[\xi^T \lambda_T^*] - CE_v[\xi^T \lambda_A^*] \geq 0$$

as a proximity measure, which has the interpretation of added monetary value, if the true model is solved instead of the assumed one.

For the MV risk-adjusted return model (9) we took the difference in objective values as a proximity measure.

3 The data-sets

The data set is the same as we have utilized in Hens and Mayer (2012) [8]. The computations have been carried out by utilizing a monthly-returns benchmark data-set with 8 indices representing asset classes; we would like to express our thanks to Dieter Niggeler of BhFS¹ for providing us with these

¹BhFS stands for Behavioral Finance Solutions, which is a spin-off company of the University of Zurich; for more information, see www.bhfs.ch.

data. This data-set is standard in computing strategic asset allocations, e.g. for pension funds or private investors.

The BhFS benchmark data-set consists of monthly net returns for 8 asset-classes (indices) for the time period 02.1994 – 05.2011, thus resulting in a sample-size of 208 elements. The indices included are listed in Table 1; a summary statistics concerning the data-set is shown in Table 2 in the Appendix.

GSCITR	Goldman Sachs Commodity Index; total return;
I3M	3 months US Dollar LIBOR interest rate;
HFRIFFM	Hedge Fund Research International, Fund of Funds, market defensive index;
MSEM	Morgan Stanley Emerging Markets Index, total return, stocks;
MXWO	MSCI World Index, total return, stocks (developed countries);
NAREIT	FTSE, US Real Estate Index, total return;
PE	LPX50, LPX Group Zurich, Listed Private Equities, total return;
JPMBD	JP Morgan Bond Index, Developed Markets , total return.

Table 1: The indices included in the monthly-returns data-set

In our numerical tests we worked with three data-sets.

3.1 First (original) data set

Since probability weighting is an important component of CPT and we wanted fully account for it, we have computed an empirical distribution (a lottery) having 15 realizations, on the basis of the BhFS benchmark data-set considered as a sample. Because we wished to avoid distributional assumptions concerning our data as far as possible, for this purpose we have utilized k -means clustering with the Manhattan-distance. Concerning the k -means clustering approach see, e.g., Everitt et al. (2011) [6] and the references therein. The resulting empirical distribution is displayed in Table 3 in the Appendix, page 19, this is our **first data-set** which we will also call as original data-set.

3.2 Second data set, involving a call option

The particular choice of $k = 15$ was motivated by our aim to get a **second data-set** by appending the original data-set by a European call option. The reason for taking also a data-set of this type is we wished to take also a data-set with a clear deviation from a normality assumption. We have appended

the original data-set a European call option on the index **MXW0**. To achieve this, we have proceeded as follows.

With a given data-set, consisting of scenarios and corresponding probabilities, we consider this as an incomplete market which should be arbitrage-free for being able to append it with a call option. For testing this we compute state prices first, by solving the following linear programming problem

$$\left. \begin{aligned} \max_{\pi, \varepsilon} \quad & \varepsilon \\ & \sum_{s=1}^S \pi_s = 1 \\ & \sum_{s=1}^S r_s^k \pi_s = r_f, \quad k = 1, \dots, K \\ & \pi_s \geq \varepsilon, \quad s = 1, \dots, S \end{aligned} \right\} \quad (11)$$

where r_f is the risk free rate. Let (π^*, ε^*) be an optimal solution. The above problem serves for computing state prices with the smallest state price being maximal. The market is arbitrage-free if for the optimal solution $\varepsilon^* > 0$ holds. Concerning state prices and incomplete markets see, e.g., Hens and Rieger (2010) [9] or Černý (2009) [3].

In our case we have $K = 8$ assets and wished to find an arbitrage-free data-set for the monthly risk-free rate $r_f = 0.002$. For achieving this we have performed computational experiments by generating empirical distributions with an increasing number of scenarios S by k -means clustering ($k = S$) as applied to our sample consisting of 208 elements. For each of these empirical distributions we have solved (11) and finally for $S = 15$ we have obtained positive state prices. This empirical distribution with $S = 15$ realizations and $K = 8$ assets (see Table 3 in the Appendix) served as the basic data-set of our empirical comparisons.

Using the state prices obtained via solving (11), we add a call option to our data-set, on the 6th index **MXW0**. The data matrix is appended by a column corresponding to the call option, with the entries scaled such that

for the added column \hat{r} the relation $\sum_{s=1}^S \hat{r}_s \pi_s^* = r_f$ holds thus guaranteeing

the arbitrage-free nature of the new data-set. As a strike price (in terms of net returns) we choose 0.1 which ensures that there is a positive payoff in only one state $s = 12$ (see Table 3) this way producing a clear deviation from

the normality assumption. Thus in the added column \hat{r} the only nonzero element is $\hat{r}_{12} = 0.283$ with the specific value computed on the basis as discussed above.

3.3 Third data set, corresponding to a normal distribution

For comparative reasons we have also generated a **third data-set** for testing the effects of a normality assumption. This data-set has been generated as follows. Taking the empirical expected value vector μ and empirical covariance matrix Σ of the original scenarios, we simulated a sample consisting of 10'000 elements from the corresponding multivariate normal distribution, by employing the standard method based on the Cholesky-factorization of Σ . Subsequently we employed k -means clustering to get 15 scenarios with corresponding probabilities.

3.4 Data set for the PT parameter settings

Concerning the parameters of the PT value function and of the probability weighting functions, we utilize the settings published by Abdellaoui, Bleichrodt and Paraschiv (2007) [1]. In Appendix C of this paper the authors present the results of an experimental elicitation of the PT-parameters for 48 subjects. The reference point is 0 throughout. Concerning loss aversion, several parameter settings are presented according to the different definitions of loss-aversion. For our numerical experiments we have chosen the classical Kahneman-Tversky loss-aversion coefficient, corresponding to the median estimator.

Regarding probability distortion, the authors present probabilities p_g and p_l for gains and losses, respectively, having the properties $w^+(p_g) = 0.5$ and $w^-(p_l) = 0.5$. For determining the parameters δ and γ in the probability distortion functions (5), the corresponding equations have to be solved for the parameters δ and γ , for fixed probability p . These equations have a unique solution for $p \geq 0.5$ while for $p < 0.5$ we have computed the solutions with the least absolute deviations in the equations. Since obviously there is no need for a high-precision solution, we employed straightforward grid search for determining δ and γ . For the case of ordinary prospect theory, we have employed the parameter γ of w^+ for the probability distortion.

All of our computations are carried out in turn for all of the 48 settings, with the subjects participating in the experiment considered as separate investors in this respect.

3.5 Determining the risk aversion parameter of investors

The Abdellaoui parameters for CPT need to be transferred to (QEU), (MV) and (CRRRA).

For this purpose we employ the lottery $\xi \sim (y, p; -x, 1 - p)$, as shown in Figure 1, with an assumed certainty equivalent of 0.

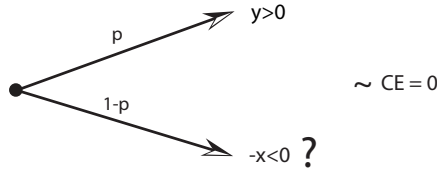


Figure 1: A standard risk tolerance question.

For investors with a CPT objective function and value function V , $CE = 0$ means that the following equation holds:

$$w^+(p)V(y) + w^-(1 - p)V(-x) = 0.$$

Taking a piecewise power value function V and assuming $RP = 0$, the explicit solution of the above equation reads:

$$x = \left[\frac{1}{\beta} \frac{w^+(p)}{w^-(1 - p)} y^{\alpha^+} \right]^{\frac{1}{\alpha^-}}.$$

Regarding investors with a quadratic utility function, who give the same response x in the above lottery, the equation for the certainty equivalent is

$$\mathbb{E}[\xi] - \kappa \mathbb{E}[\xi^2] = 0$$

with the explicit solution for the risk aversion parameter κ :

$$\kappa = \frac{py - (1 - p)x}{py^2 + (1 - p)x^2}.$$

For investors with a MV risk-adjusted return objective function, we solve the linear equation

$$\mathbb{E}[\xi] - \alpha \sigma^2[\xi] = 0$$

for α .

Finally, for the CRRA power function investors we solve the nonlinear equation

$$p(1+y)^\theta + (1-p)(1-x)^\theta = 1.$$

Notice that $\theta = 0$ is an obvious solution; we are looking for a further, different solution. For the purpose of solving the above equation we utilized the *FindRoot* procedure of *Mathematica*®.

The PT parameter values of the 48 investors we have chosen according to Abdellaoui et al. [1] and we took $p = 0.33$ in accordance with [1]. The parameters δ and γ of the probability weighting function we computed as described in Hens and Mayer (2012) [8] and we have set $x = 0.3$ in terms of net returns. Then we computed the risk aversion parameters κ , α and θ as described above. The results are displayed in Table 4.

4 Comparing the optimal portfolios

In this section we report our results regarding the differences between the optimal portfolios, obtained from the different portfolio selection approaches as formulated in (6), (8), (9) and (10), with the optimal portfolios denoted by λ_{CPT}^* , λ_Q^* , λ_{MV}^* and λ_{CRR}^* , respectively.

In our paper Hens and Mayer (2012) [8] we compared λ_{CPT}^* and λ_{CPTMV}^* , where λ_{CPTMV}^* is the portfolio by maximizing the CPT objective function along the MV-frontier. We found that for the original data set the portfolios differ substantially; for the data set with the call option the difference increases noticeably, whereas for the data set with the normal distribution the difference reduces to a large extent.

In this paper we report our results regarding the comparison of λ_Q^* , λ_{MV}^* , λ_{CRR}^* and λ_{CPT}^* . The numerical study was carried out by employing the same data sets as in [8]; for a detailed description see Section 3 above.

The optimal portfolios, corresponding to the different models and data-sets, are displayed in Figures 4, 5 and 6, in the Appendix.

Regarding the data-set with the call option, we observe that none of the QEU and MV investors invest any fraction of their wealth in the call option, see Figure 5. Thus the optimal portfolios for the QEU and MV investors are the same for the original data set and for the data set with the added call option. The same is true for the CRR investors with a sole exception: investor S25 invests a positive but negligible fraction of 0.002 in the call option.

In contrast, CPT investors invest significant amounts in the call option; 24 out of them invest even their whole wealth in that asset, see Figure 5. This phenomenon is called *skewness loving* of CPT investors and has been analyzed by Barberis and Huang (2008) [2]. They assume a normal distribution of the risky assets and add a small, independent and positively skewed security to these assets. Our numerical results indicate that the skewness loving behavior of CPT investors extends far beyond the theoretical framework, for which it was analyzed. For the details see our paper Hens and Mayer (2012) [8].

The comparative numerical results regarding the original data-set are displayed in Table 5. We observe that the differences between the CPT portfolios and the portfolios originating from the QEU and MV approaches are considerable. The difference between the QEU and MV portfolios is quite small, but nevertheless it is not zero (2 BP's in annual terms).

Regarding the data-set with the call option added, the results are shown in Table 6. As mentioned above, the QEU and MV investors have the same optimal portfolios as for the original data-set, since none of them invests into the call option. Thus the changes in the table only involve the positions with CPT involved. Comparing these with the results for the original data-set, we observe that the distances have increased substantially.

For the data set with the normal approximation, the results can be seen in Table 7. Our observations are quite similar to those concerning the original data-set. Thus, in the comparisons aspect, normality does not imply dramatic changes in the CE distances; in general a slight decrease can be observed.

For the quadratic utility function it is important to ensure that none of the portfolio return realizations are beyond the satiation point. We have checked this for all of the optimal QEU portfolios and observed that in fact none of the realizations of these portfolios was beyond the satiation point of the quadratic utility function.

Levy and Levy (2004) [12] demonstrated that for normally distributed returns the CPT portfolios are located along the mean–variance frontier. Pirvu and Schulze (2012) [14] generalized this result for the class of elliptically symmetric distributions. Regarding our data set constructed on the basis of a normal approximation, this phenomenon can be seen in Figure 2. In the parameter data–set there are 11 investors, who have a negative κ values. Dropping these investors, Figure 3 results.

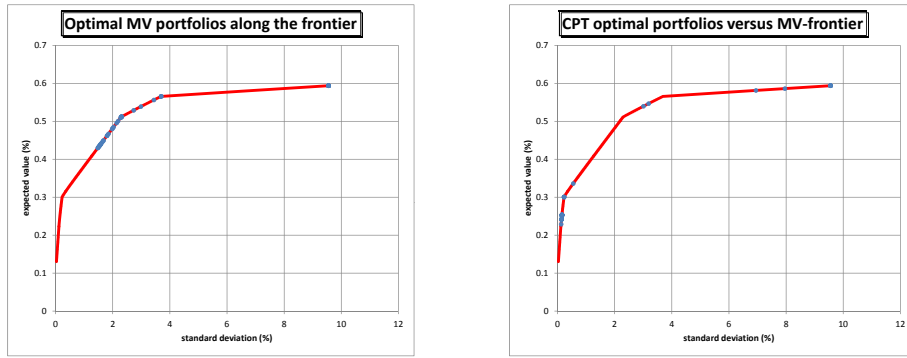


Figure 2: Data set with the normal distribution: On the left hand side the $\sigma - \mu$ values of λ_Q^* portfolios and on the right–hand side the $\sigma - \mu$ values of the λ_{CPT}^* portfolios are displayed.

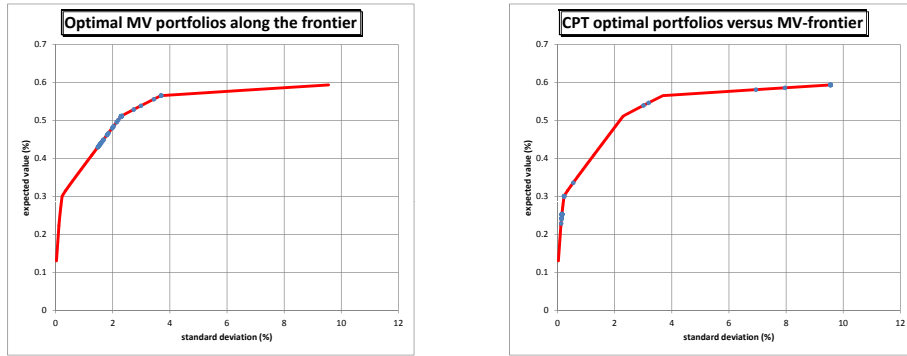


Figure 3: Data set with the normal distribution: On the left hand side the $\sigma - \mu$ values of λ_Q^* are displayed now only for the risk averse investors, and on the right–hand side the $\sigma - \mu$ values of the λ_{CPT}^* portfolios are displayed.

5 Conclusion

This paper has shown that the choice of the decision theory with which a client is modeled is crucial for the investment advice given. For the same market data and the same answer of a client to a standard risk-profiler question the recommended portfolios differ (measured in certainty equivalents) by 2% to 6% p.a. To avoid giving the wrong advice the financial adviser has to first figure out which decision model describes best the client. Thus standard risk profiler questions should be extended to question like those in Kahneman and Tversky (1979) [10] which reveal which type of decision maker a client is. Assuming one decision model without good justification can be fatal. Further research has to figure out which combination of questions are best suited in a risk-profiler. This research is very important for financial advisors trying to improve their services and for regulators trying to avoid major mistakes in financial advice.

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6 Appendix: Tables and Figures

	GSCITR	HFRIFFM	I3M	JPMBD	MSEM	MXWO	NAREIT	PE
Min.	-3.7205	-2.3395	-0.0885	-0.4539	-5.1791	-2.1162	-3.0722	-3.1710
Mean	0.0441	0.0584	0.0149	0.0364	0.0669	0.0669	0.0546	0.0594
Median	0.0909	0.1208	0.0065	0.0303	0.0872	0.1259	0.1402	0.1283
Max.	2.3879	1.3723	0.1839	0.5459	3.8823	1.6683	2.2549	3.2807
Std. Dev.	0.8279	0.4503	0.0762	0.1732	1.2644	0.5335	0.6409	0.7517
Skewness	-0.4903	-1.8369	0.4415	0.0302	-0.5283	-0.6643	-0.8683	-0.2963
Kurtosis	4.5874	10.604	2.1578	3.5438	4.7248	4.8595	7.6889	8.0357

Table 2: Summary statistics for the monthly-returns data-set. The statistics is for annualized data, for facilitating comparisons based on the I_{CED} proximity index.

P	GSCITR	HFRIFFM	I3M	JPMBD	MSEM	MXWO	NAREIT	PE	Call on MXWO
0.1731	0.0441	0.0168	0.0012	0.0060	0.0484	0.0268	0.0315	0.0191	0
0.1635	-0.0024	0.0041	0.0023	0.0062	-0.0157	0.0023	0.0107	0.0109	0
0.1490	-0.0384	0.0213	0.0002	-0.0012	0.0938	0.0318	0.0205	0.0271	0
0.1346	0.0756	0.0098	0.0009	0.0030	-0.0175	-0.0095	-0.0213	-0.0044	0
0.1010	-0.0816	-0.0025	0.0007	-0.0023	-0.0294	-0.0082	-0.0052	-0.0174	0
0.0865	0.0654	0.0350	-0.0023	0.0056	0.1353	0.0500	0.04918	0.0682	0
0.0865	0.0014	-0.0261	0.0060	0.0008	-0.1334	-0.0518	-0.0401	-0.0691	0
0.0385	-0.0284	-0.0117	0.0051	0.0023	-0.1663	-0.0074	-0.0354	0.0013	0
0.0241	-0.1388	-0.0376	-0.0029	-0.0038	-0.1643	-0.1059	-0.1447	-0.1658	0
0.0096	0.0370	0.0634	-0.0019	-0.0024	0.1929	0.0840	0.0978	0.2623	0
0.0096	-0.0574	-0.0743	-0.0026	0.0266	0.1505	0.0933	0.1499	0.0562	0
0.0096	0.1482	0.0240	-0.0058	0.0168	0.3165	0.1232	0.1467	0.1585	0.2832
0.0048	-0.3100	-0.1935	0.0064	-0.0113	-0.3817	-0.1763	-0.2560	-0.2643	0
0.0048	-0.1459	-0.1950	0.0080	-0.0035	-0.2370	-0.1050	-0.0862	-0.1193	0
0.0048	-0.0314	-0.1313	0.0005	0.0252	-0.4316	-0.1259	-0.0833	-0.1161	0

Table 3: The empirical distribution, constructed via k -means clustering ($k = 15$) from the monthly returns data-set with the last column corresponding to the appended European call-option on MXWO.

	α^+	α^-	β	γ	δ	loss: $-x$ (lottery)	κ (quad util.)	α (MV)	θ (CRRA)
S1	1.03	0.7	4.11	0.57	0.86	-0.01	3.14	4.43	-46.8
S2	1.20	1.24	1.08	0.61	0.64	-0.20	-0.61	-0.6	2.22
S3	1.02	0.60	2.25	0.71	0.71	-0.01	2.99	4.08	-27.5
S4	0.85	0.80	2.75	0.65	0.68	-0.04	2.28	2.72	-8.34
S5	0.57	0.85	0.85	1.00	0.58	-0.34	-1.20	-1.41	3.59
S6	0.56	1.01	2.34	0.89	0.58	-0.15	-0.10	-0.10	1.21
S7	0.96	0.79	1.85	0.65	0.58	-0.07	1.63	1.79	-3.90
S8	2.17	1.09	6.80	0.64	0.64	-0.01	3.09	4.33	-38.6
S9	1.28	0.66	2.08	0.53	0.89	-0.01	3.09	4.32	-38.3
S10	1.02	0.70	1.28	0.54	0.71	-0.05	2.06	2.38	-6.47
S11	0.72	0.68	1.72	0.91	0.65	-0.06	1.75	1.94	-4.47
S12	0.74	0.91	1.20	0.61	0.57	-0.21	-0.67	-0.69	2.34
S13	0.72	0.67	1.39	0.68	0.68	-0.08	1.29	1.37	-2.48
S14	1.93	3.07	0.30	0.50	0.53	-0.60	-1.11	-1.69	4.17
S15	0.59	0.82	1.53	0.71	0.71	-0.14	0.19	0.19	0.59
S16	0.61	0.82	2.01	0.81	0.60	-0.11	0.64	0.65	-0.48
S17	1.00	0.85	3.06	0.56	0.52	-0.04	2.23	2.64	-7.84
S18	0.68	0.82	1.63	0.86	0.81	-0.10	0.88	0.90	-1.13
S19	0.96	0.80	1.54	0.62	0.60	-0.08	1.27	1.34	-2.39
S20	0.90	0.71	7.01	0.81	0.75	-0.01	3.18	4.55	-60.3
S21	0.87	1.02	1.93	0.71	0.63	-0.12	0.39	0.39	0.14
S22	0.69	0.66	1.13	0.75	0.68	-0.12	0.55	0.55	-0.23
S23	0.84	0.55	1.70	0.75	0.68	-0.03	2.71	3.48	-15.0
S24	0.50	0.54	1.57	0.61	0.63	-0.06	1.83	2.05	-4.94
S25	0.61	0.33	2.53	0.67	0.63	-0.01	3.29	4.86	-221.
S26	1.11	0.75	2.94	0.78	0.83	-0.02	2.90	3.87	-21.6
S27	0.38	0.48	0.87	0.48	0.97	-0.09	1.15	1.21	-1.99
S28	0.51	0.51	4.35	0.64	0.85	-0.01	3.22	4.65	-80.7
S29	0.87	0.75	2.06	0.63	0.63	-0.05	2.02	2.31	-6.12
S30	1.24	0.66	0.63	0.63	0.86	-0.08	1.33	1.41	-2.60
S31	2.05	0.71	1.92	0.68	0.71	-0.01	3.20	4.59	-66.4
S32	0.49	1.01	0.41	0.91	0.61	-0.89	-0.89	-1.59	4.23
S33	0.72	1.08	1.46	0.68	0.58	-0.23	-0.83	-0.86	2.65
S34	0.46	0.80	1.06	0.61	0.65	-0.25	-0.97	-1.05	2.97
S35	0.71	0.60	3.77	0.96	0.64	-0.01	3.05	4.22	-33.0
S36	0.56	0.58	2.05	0.86	0.73	-0.04	2.44	2.98	-10.1
S37	0.86	0.81	4.32	0.59	0.56	-0.03	2.63	3.33	-13.3
S38	1.32	0.74	1.57	0.64	0.91	-0.03	2.70	3.45	-14.7
S39	0.66	0.60	1.76	0.55	0.60	-0.05	2.14	2.50	-7.09
S40	0.71	0.59	1.68	0.86	0.83	-0.04	2.46	3.02	-10.9
S41	0.50	0.51	0.87	0.58	0.64	-0.15	-0.09	-0.09	1.19
S42	0.69	0.66	1.14	0.58	0.59	-0.12	0.43	0.43	0.05
S43	0.67	0.81	1.66	0.65	0.68	-0.11	0.71	0.73	-0.67
S44	0.60	0.62	2.41	0.65	0.62	-0.04	2.40	2.91	-9.65
S45	0.51	1.57	0.40	0.54	0.91	-0.76	-0.98	-1.65	4.22
S46	0.63	0.50	2.37	0.52	0.55	-0.02	2.93	3.94	-23.3
S47	0.66	0.87	0.80	0.81	1.00	-0.25	-0.94	-1.00	2.88
S48	1.35	0.43	1.64	0.68	0.63	-0.003	3.27	4.80	-149.

Table 4: The CPT parameter values from Abdellaoui et al. [1], the computed γ and δ values, the computed losses $-x$ in the lottery as well as the corresponding κ , α and θ parameters in the alternative objective functions.

	assumed type							
	MV		quad. util.		CRRA		CPT	
MV	0		min	0.00	min	0.00	min	0.00
			mean	0.02	mean	0.13	mean	2.72
			median	0.01	median	0.10	median	1.96
			max	0.06	max	1.17	max	10.7
quad. util.	min	0.00	0		min	0.00	min	0.00
	mean	0.01			mean	0.19	mean	2.65
	median	0.01			median	0.15	median	2.04
	max	0.04			max	1.32	max	10.4
CRRA	min	0.00	min	0.00	0		min	0.00
	mean	2.21	mean	4.09			mean	2.35
	median	0.18	median	0.29			median	1.64
	max	39.9	max	60.6			max	10.3
CPT	min	0.00	min	0.00	min	0.00	0	
	mean	2.59	mean	2.87	mean	1.90		
	median	1.51	median	1.82	median	1.07		
	max	17.2	max	17.2	max	17.2		

Table 5: Original data set: Distances between the portfolios measured in terms of CE, expressed in percents and showing annualized values. Columns correspond to the assumed type of investors, whereas rows represent the actual type.

	assumed type							
	MV		quad. util.		CRRA		CPT	
MV	0		min	0.00	min	0.00	min	0.00
			mean	0.02	mean	0.13	mean	4.03
			median	0.01	median	0.10	median	3.41
			max	0.06	max	1.17	max	11.2
quad. util.	min	0.00	0		min	0.00	min	0.00
	mean	0.01			mean	0.19	mean	3.60
	median	0.01			median	0.15	median	3.08
	max	0.04			max	1.32	max	10.7
CRRA	min	0.00	min	0.00	0		min	0.00
	mean	2.21	mean	4.09			mean	3.26
	median	0.18	median	0.29			median	2.26
	max	39.9	max	60.6			max	10.5
CPT	min	0.00	min	0.00	min	0.00	0	
	mean	13.5	mean	13.7	mean	12.8		
	median	6.23	median	6.28	median	5.56		
	max	89.3	max	91.6	max	84.9		

Table 6: Call option added: Distances between the portfolios measured in terms of CE, expressed in percents and showing annualized values. Columns correspond to the assumed type of investors, whereas rows represent the actual type.

	assumed type							
	MV		quad. util.		CRRA		CPT	
MV	0		min	0.00	min	0.00	min	0.00
			mean	0.03	mean	0.16	mean	1.49
			median	0.00	median	0.12	median	1.37
			max	0.11	max	0.46	max	5.23
quad. util.	min	0.00	0		min	0.00	min	0.00
	mean	0.24			mean	0.25	mean	1.60
	median	0.00			median	0.17	median	1.57
	max	0.08			max	0.72	max	5.23
CRRA	min	0.00	min	0.00	0		min	0.00
	mean	0.94	mean	1.90			mean	1.18
	median	0.10	median	0.10			median	0.91
	max	10.6	max	20.3			max	5.26
CPT	min	0.00	min	0.00	min	0.00	0	
	mean	3.19	mean	3.56	mean	2.40		
	median	2.94	median	3.14	median	2.57		
	max	9.23	max	11.7	max	6.36		

Table 7: Normal distribution: Distances between the portfolios measured in terms of CE, expressed in percents and showing annualized values. Columns correspond to the assumed type of investors, whereas rows represent the actual type.

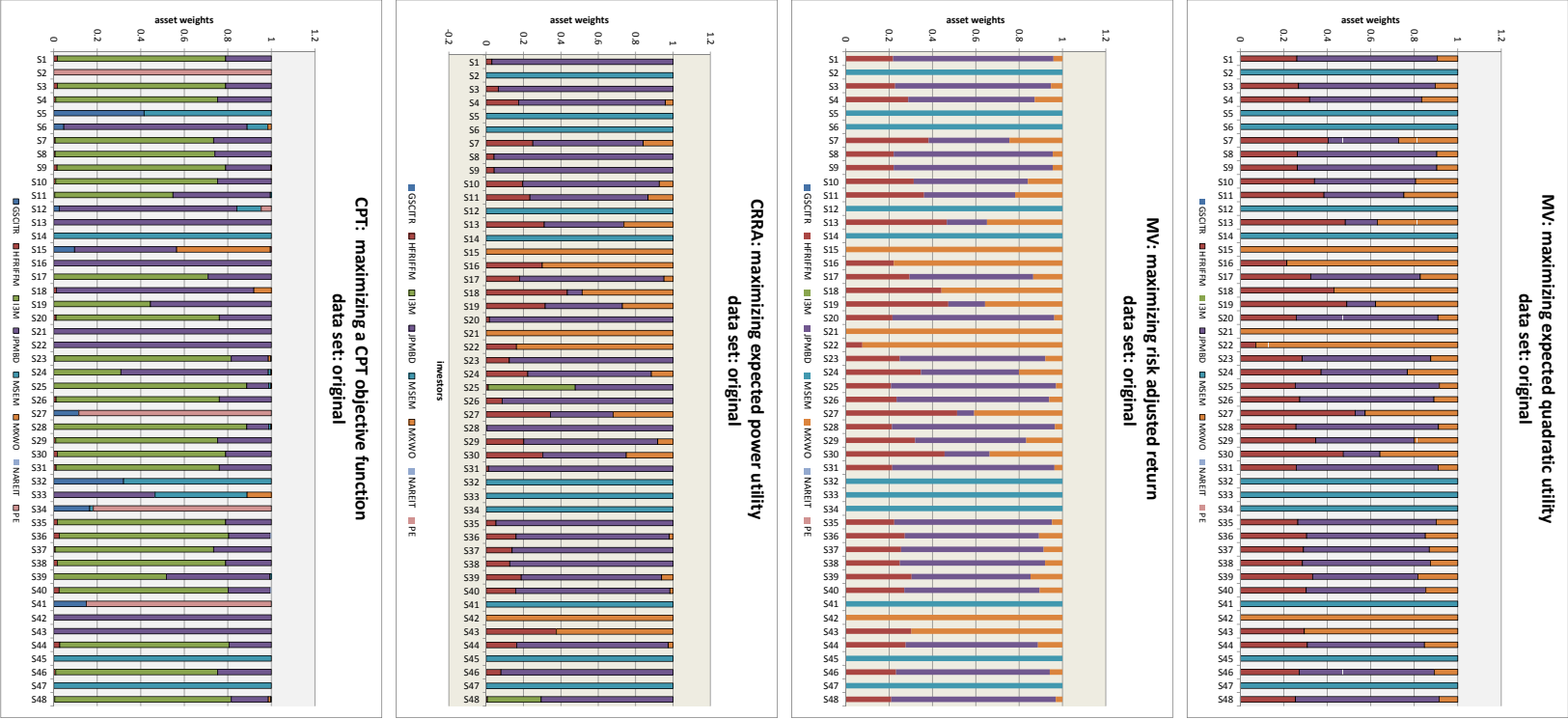


Figure 4: Original data-set; portfolios selected according to the four different portfolio selection approaches.

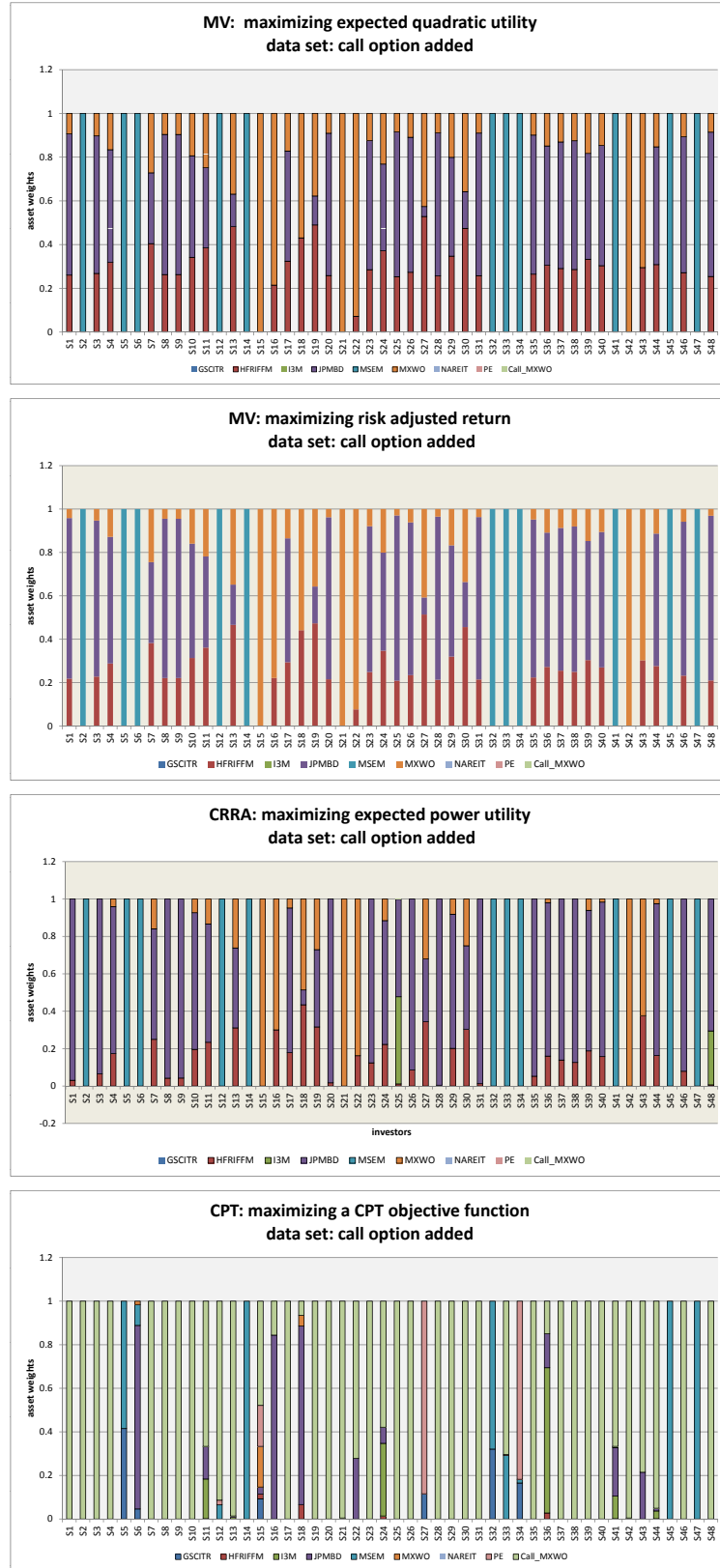


Figure 5: Call option added; portfolios selected according to the four different portfolio selection approaches. Notice that for the first three approaches the same portfolios were obtained as for original data-set.

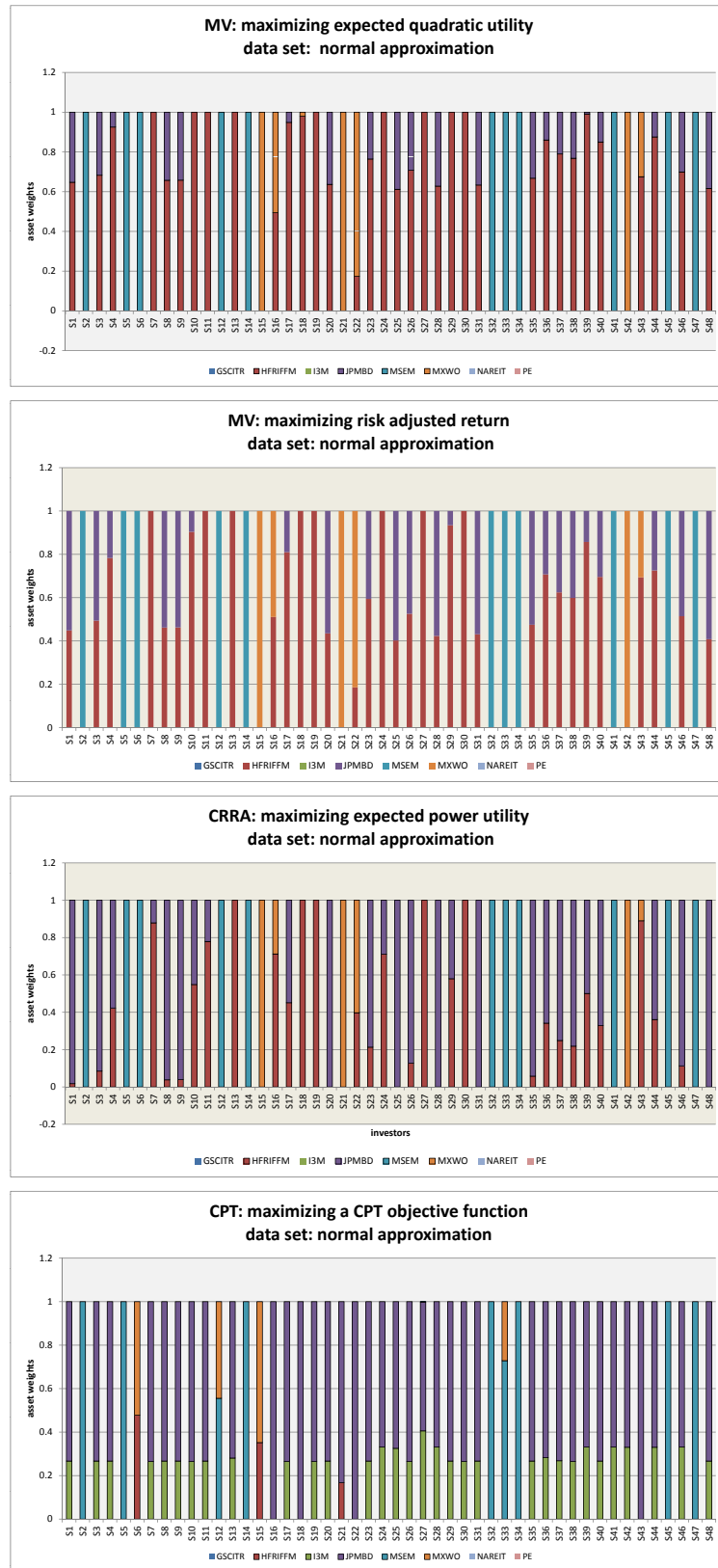


Figure 6: Normal distribution; portfolios selected according to the four different portfolio selection approaches.